MTL106 Probability and Stochastic Processes

Sem-II 2016-17

TUTORIAL-2 Random Variables

Solutions

1. X is a random variable if X: Ω -> R and X-1(-] ε F ∀ x ε R where F is given sigma field

Define a real valued function X such that

X(0) = 0

X(1) = 0

X(2) = 1

Now X-1(-,0] = {0,1} ∉ F so X is real valued function and not a random variable

1. For a function F(x) to be a distribution function, it must satisfy four properties:
2. 0≤F(x)≤1
3. It should be monotonically increasing function
4. It should be right continuous

‘a’ is satisfied since F(x) always takes values between 0 and 1

‘b’ is also satisfied

At x = ½ value of function is ½ while its right hand limit is 1 so property is ‘c’ is violated

Hence it is not a distribution function

1. F(x) = ()tan-1x,

As tan-1(x) takes value from -π/2 to π/2

given function takes values from -1/2 to ½ so property ‘a’ is violated so it is not a distribution function

‘a’: F(x) is 0 for x < 0 and between 0 & 1 for x > 0 so ‘a’ is satisfied

‘b’: at x = 0, it takes value 0 and as x increases, , hence F(x) is monotonically increasing function

‘c’: only point where function definition changes is x = 0 where value of function is equal to its right hand limit and when x > 0, F is continuous so F(x) is right continuous

‘d’: As x → -, F(x) = 0 and as x →

Hence F is distribution function

1. a. P(X = 10) = F(10) – F(9) = 1 – 0.998 = 0.002

b. P(X ≤ 5 / X > 2) = P(2 < X ≤ 5)/P(X > 2) = (F(5) – F(2))/(1 – P(X ≤ 2))

= 0.074/0.102 = 0.7254

1. pX(x) = αpx, x = 0, 1, 2, …

for it to be a probability mass function

αp0 + αp1 + αp2 + … + = 1

(for sum to converge, |p| < 1)

α + p = 1 ------- (i)

Also pX(x) > 0 which is possible when both and p are positive or when both are negative

Since their sum is 1, both cannot be negative so both are positive

α > 0 and p > 0

This means α > 0 and 0 < p < 1 -------- (ii)

P(X > a + s / X > a) = α()/α()

=

P(X ≥ s) = α() = α = ( Using (i) )

Hence it satisfies memoryless property

Given P(X=2) = ¼

1. Since x=2 is the only jump discontinuity, function must be continuous at x = 4

¾ =

Also, jump at x=2 is because of probability at x=2 since given random variable is of mixed type

P(X = 2) = ¾ - α(x+3) = ¼

5α = ½, α = 1/10

1. P(X < 3 / X ≥ 2) = P(2 ≤ X < 3)/P(X ≥ 2)

In the range (2,3), since FX(x) is constant, P(2 ≤ X < 3) = P(X = 2) = ¼

P(X ≥ 2) = 1 – P(X < 2) = 1 - 5α = 1 – ½ = ½ (using definition of FX)

Required probability =

1. Let X(in minutes) be a random variable denoting time after 6:00AM when the bus arrives

X ~ U(0,20)

Cdf of X is FX(x) =

1. P(X > 5) = 1 – P(X ≤ 5) = 1 – F(5) = 1 – ¼ = ¾
2. P(X > 15/X > 10) = P(X > 15)/P(X > 10) =

=

1. Accidents follow poisson process with parameter λ = 9/30 days-1

This means that number of accidents till time t days (Nt) follows poisson distribution with parameter λt

1. Number of accidents in first 15 days be denoted by N15

N15 ~ Poi(15.λ)

P(N15 = 4) =

1. Let Nt denote random variable denoting number accidents in t days

We know that Nt ~ Poi(λt)

P(all 4 accidents occurred in last 7 days / 4 accidents occurred in fisrt 15 days)

= P(no accidents in first 8 days(N8) and 4 accidents in next 7 days(N7))/P(N15 = 4)

(Since N8 and N7 independent of each other i.e. number of accidents in last 7 days is independent of number of accidents in first 8 days)

=

1. Nt = N1t + N2t + … + Nnt

Where Nit = ∀ i = 0, 1, 2, …, n

P(Nit=1) = P(time to faiure of ith > t)

(let time to failure be a random variable X ~ exp(λ))

= 1 – P(time to failure ≤ t)

= 1 – (1 - )

=

Nt can take values k = 0, 1, 2, …, n(maximum number of units)

P(Nt = k) = (choose k units out of n and multiply probability of them being in operation at time t with probability of remaining units not being operational at time t)

Where, k = 0, 1, 2, …, n

1. f and g are density functions

property of density function is

1. is λf + (1 - λ)g a density function where λ is a constant

=

=

=

= 1

So it is a density function

1. is fg a density function

(fg)(x) = f(x)g(x)

We can disprove this by constructing a counter example

f and g denotes U(0,1) and U(2,3) density functions respectively but their product is zero everywhere since there are no common values for which both of them are 1

Since a function h(x) = (fg)(x) = 0 cannot be a density function, fg is not a density function always

X is mixed type random variable since there are jump discontinuities in its cdf

At x = 1, FX(x)=1/25 while

Similarly we can see there are jump discontinuities at x = 2,3 by comparing left hand limit and value of function at these points

To decompose the cdf of X into a linear combination of a discrete cdf and continuous cdf we first write these cdf’s individually.

Discrete Part,

P(X = 1) = 1/25 – 0 = 1/25

P(X = 2) =

P(X) = 3 = 1 –

FD’(x) =

Since value of FD’(x) as x → is not 1, we need to multiply it with a normalizing factor α’ to make it a valid cdf

and get FD(x) = α’ FD’(x)

As x →

Thus FD(x) =

Similarly we can find FC’(x) that is continuous part of cdf by subtracting FD’(x) from original cdf F(x) and then normalizing it as above to get a valid continuous cdf

FC’(x) =

FC(x) = β’ FC’(x), here a normalizing factor has been multiplied with FC’(x) so that its value becomes 1 as x →

Again value of the normalizing factor β’ = 2

Thus FC(x) =

Now we need to express original cdf FX(x) as a linear combination of FD(x) and FC(x)

FX(x) = αFd(x) + βFc(x)

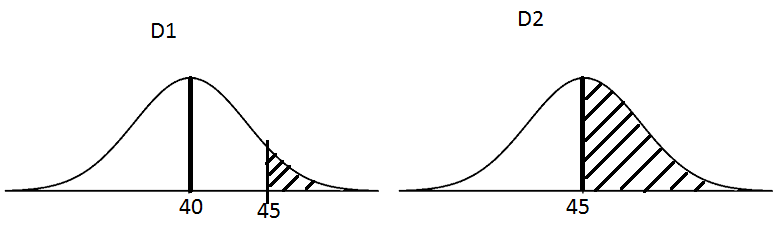
From observation and also the way we have constructed FD(x) and FC(x), we can see α = ½ and β = ½

P(X > 30 / X > 20) =

= =

1. D1 ~ N(40,36) and D2 ~ N(45, 9 )
2. If device has to be used for 45 hours we will select the device that has higher probability of lasting more than 45 hours

P(lasting more than 45 hours for device) = area under the pdf curve to the right of 45



As we can see it(shaded region) is more for D2, it is preferable

1. If device has to be used for 42 hours then also D2 is preferable as can be seen from the following plot:

